

⊗ Class 12 Mathematics – Chapter: Relations and Functions

1. Introduction

- Relations and functions extend the idea of mapping elements from one set to another.
- Fundamental in understanding mathematical models.

2. Relations

- A relation from set A to set B is a subset of the Cartesian product $A \times B$.
- If $(a,b) \in R$, then a is related to b .
- **Domain:** Set of all first elements.
- **Range:** Set of all second elements.

3. Types of Relations

- **Reflexive:** $(a,a) \in R$ for all $a \in A$.

- **Symmetric:** If $(a,b) \in R(ab) \in R(a,b) \in R$, then $(b,a) \in R(ba) \in R(b,a) \in R$.
- **Transitive:** If $(a,b) \in R(ab) \in R(a,b) \in R$ and $(b,c) \in R(bc) \in R(b,c) \in R$, then $(a,c) \in R(ac) \in R(a,c) \in R$.
- **Equivalence relation:** Relation that is reflexive, symmetric, and transitive.

4. Functions

- A function f from A to B assigns each $a \in A$ exactly one $b \in B$.
- Notation: $f:A \rightarrow B$.
- **Domain:** Set A .
- **Co-domain:** Set B .
- **Range:** Subset of B consisting of actual images.

5. Types of Functions

- **One-to-one (Injective):** Distinct elements in domain map to distinct elements in range.
- **Onto (Surjective):** Every element of co-domain is mapped to by some element of domain.

- **Bijective:** Both one-to-one and onto.

6. Composition of Functions

- If $f:A \rightarrow B$ and $g:B \rightarrow C$, then $g \circ f: A \rightarrow C$.
- Defined as $(g \circ f)(x) = g(f(x))$.

7. Inverse of a Function

- For bijective function f , inverse f^{-1} exists.
- Satisfies $f^{-1}(f(x)) = x$.

8. Graph of a Function

- Set of points $\{(x, f(x)) \mid x \in \text{domain}\} \setminus \{(x, f(x)) \mid x \in \text{domain}\}$.
- Vertical line test to check if a curve is a function.

9. Applications

- Modeling relationships between variables.
- Used in calculus, algebra, and applied sciences.

10. Exam Tips

- Understand definitions and properties.
- Practice proving properties of relations.
- Learn to identify function types.
- Solve problems on composition and inverses.